1. Convolution

$$\begin{matrix} 1 & 1 & 2 & 2 \\ 1 & 1 & 2 & 2 \\ 1 & 1 & 2 & 2 \\ 1 & 1 & 2 & 2 \end{matrix} \quad * \quad \begin{matrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{matrix} \quad =$$

1	2	4	5	4	2
2	5	9	12	10	4
3	7	13	17	14	6
3	7	13	17	14	6
2	5	9	12	10	4
1	2	4	5	4	2

2. Gaussian

- The Fourier transform of a Gaussian function $e^{-\pi x^2}$ is given by

$$F(s) = \int_{-\infty}^{\infty} e^{-\pi x^2} e^{-i2\pi xs} dx$$

$$= \int_{-\infty}^{\infty} e^{-\pi(x^2+2ixs)} dx$$

$$= \int_{-\infty}^{\infty} e^{-\pi s^2} e^{-\pi(x^2+2ixs-s^2)} dx$$

$$= e^{-\pi s^2} \int_{-\infty}^{\infty} e^{-\pi(x+is)^2} dx; \quad [y \leftarrow x+is, dy/dx = 1]$$

$$= e^{-\pi s^2} \int_{-\infty}^{\infty} e^{-\pi y^2} dy$$

Note that the integral in the last equation = 1, so that a Gaussian transforms to another Gaussian.